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Using realistic mathematics education and the DAPIC problem-solving process to enhance secondary school students' mathematical literacy

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A B S T R A C T

Mathematical literacy plays an important role as one of life skills. It is a fundamental skill which is as necessary as literacy. Therefore, mathematics teaching in schools must aim to develop mathematical literacy and to enhance each students' ability to use and apply mathematical knowledge in order to solve real life problems or situations. According to Realistic Mathematics Education, real world problems are used as a source or a starting point for learning and developing mathematical concepts. Students should have the opportunity to build their own mathematical knowledge through the teacher's guidance. The DAPIC problem-solving process consists of five elements which make up its acronym, namely (1) define, (2) assess, (3) plan, (4) implement, and (5) communicate. Realistic mathematics education and the DAPIC problem-solving process should be collaboratively used to develop students' mathematical literacy.

This study was based on research and development design. The main purposes of this study were to develop an instructional process for enhancing mathematical literacy among students in secondary school and to study the effects of the developed instructional process on mathematical literacy. The instructional process was developed by analyzing and synthesizing realistic mathematics education and the DAPIC problem-solving process. The developed instructional process was verified by experts and was trialed. The designated pre-test/post-test control method was used to study the effectiveness of the developed instructional process on mathematical literacy. The sample consisted of 104 ninth grade students from a secondary school in Bangkok, Thailand. The developed instructional process consisted of five steps, namely (1) posing real life problems, (2) solving problems individually or in a group, (3) presenting and discussing, (4) developing formal mathematics, and (5) applying knowledge. The mathematical literacy of the experimental group was significantly higher after being taught through the instructional process. The same results were obtained when comparing the results of the experimental group with the control group.

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Introduction

Mathematical literacy is important. According to Devlin (2000, p. 24) and Watson (2002, p. 157), mathematical literacy is a fundamental skill as necessary as literacy. Watson (2002, p. 157) and Steen, Turner, and Burkhardt (2007, p. 286) also stated that mathematical literacy was one of the key objectives of instructional organization in schools. Mathematics teaching in schools aimed to provide students with mathematical literacy—an ability to use and apply mathematical knowledge in real life situations happening outside schools. Mathematical literacy has a unique characteristic which is different from substantive mathematics. According to De Lange (2003, p. 80), mathematics in schools focused on the substantive content, while mathematical literacy focused on how to use mathematics in real life.

In Thailand, although the importance of mathematics instructional organization has been recognized, there were several problems found in terms of mathematics instruction regarding the results of both national and international mathematics tests, as follows:

1) Students forgot the mathematics knowledge they had previously learned. They could not recall, understand, or recognize the importance of mathematical knowledge. They also believed that mathematics was not related to their everyday life and could not apply it to their real life (Plangprasobchoke, Boonprajak, & Phuudom, 2008);

2) The results of the Thailand Ordinary National Educational Test found that the average mathematics scores of Thai ninth grade students were below 50 percent, year after year (The National Institute of Educational Testing Service, Thailand, 2010);

3) According to the Programme for International Student Assessment (PISA) organized by the Organization for Economic Cooperation and Development (OECD), Thai students' average scores for mathematical literacy in 2000, 2003, 2006, and 2009 were 432, 417, 417, and 419 points, respectively. These scores were below the average scores of OECD of 500 points in 2000, 2003, and 2006, and 496 points in 2009 (OECD, 2004, 2007, 2010). These evaluation results show the lack of quality of Thai students and mathematics instruction.

According to the aforementioned importance and problems, it was necessary to intensively develop and enhance students' mathematical literacy. Teachers play an important role in empowering students' mathematical experience in order to further apply mathematics to their real life. Martin (2007, p. 30) also stated that mathematical illiteracy was not the result of the teaching contents but resulted from the instructional methods which were applied by teachers. The traditional instructional methods, including memorization of mathematics rules or formulas which were not related to students' real life or experience, could not enhance students' mathematical literacy. Therefore, in order to develop and enhance students' mathematical literacy, it was necessary to seek a better method or instructional process.

As stated above, the researcher was therefore interested in developing an instructional process to enhance the mathematical literacy of secondary school students as a guideline to develop students' mathematical literacy.

Purposes of the Study

This study was based on research and development design. The main purposes of this study were: (1) to develop an instructional process for enhancing mathematical literacy among students in secondary school; and (2) to study the effects of the developed instructional process on mathematical literacy.

Definition of Mathematical Literacy

Mathematical literacy refers to students' knowledge and ability to take and apply the mathematical knowledge and skills gained from classes to their real life experience and understand the situations involving mathematics. Moreover, it includes the ability to consider ‘when’ and ‘how’ to apply such mathematical knowledge. Mathematical literacy consists of the following two components.

1. **Knowledge** refers to conceptual and procedural knowledge that is essentially fundamental to connect and solve mathematical problems encountered in real life.
    - 1.1 **Conceptual knowledge** refers to knowledge about facts, meanings, constructions, ideas, principles, laws, formulas, and concepts about mathematical topics.
    - 1.2 **Procedural knowledge** refers to knowledge on how to use mathematical procedures, languages and symbols, and interpreting and drawing graphs and tables.
2. **Competency** refers to students' ability to apply the mathematical knowledge and skills gained from the classroom to their real life and to understand the situations involving mathematics. It also consists of the following abilities: (1) understanding problems, (2) selecting knowledge, (3) outlining the plan, (4) solving and reasoning, and (5) examining the solutions.

Literature Review

Realistic Mathematics Education

Realistic mathematics education is based on the idea of Freudenthal and his colleagues at the Freudenthal Institute (Van den Heuvel-Panhuizen, 2000, p. 3). Instead of viewing mathematics as a subject for transmission, Freudenthal stated the idea of mathematics as a human activity. Mathematics had to be connected to reality, stay close to children's experiences and be relevant to society. Mathematics lessons should give students opportunity to ‘reinvent’ mathematics by doing (Van den Heuvel-Panhuizen, 2000, p. 3). This meant that in mathematics education, the focal point should not view mathematics as a closed system but rather it should be viewed as the process of mathematisation (Freudenthal, 1968 as cited in Van den Heuvel-Panhuizen, 2000, p. 3).

According to the theory of Realistic Mathematics Education, the real world is a source or a starting point for the development of mathematical concepts (Freudenthal, 1991 as cited in Doorman et al., 2007, p. 406). Well chosen contextual problems offer opportunities for the students to develop informal and highly context-specific solution strategies that are used to support mathematical concept building (Gravemeijer & Doorman, 1999 as cited in Doorman et al., 2007, p. 406). Mathematics education is organized as a process of guided reinvention, where students can experience a process in the same way as it was invented (Gravemeijer, 1997, p. 322).

The three key principles of realistic mathematics education could be described as follows.

1. Guided reinvention: Students should be given an opportunity to experience a process similar to the process in which mathematics was invented. The history of mathematics could be used as a source of inspiration. During the learning process, students should have an opportunity to build their own mathematical knowledge. Students' informal strategies could be interpreted as anticipated more formal procedures. Contextual problems allowing a wide variety of solution procedures should be selected, and preferably solution procedures could reflect a possible learning route by itself (Gravemeijer, 1997, pp. 328–342; Gravemeijer & Terwel, 2000, pp. 786–788).

2. Didactical phenomenology: Situations where a given mathematical topic is applied required investigation to reveal the sort of applications that have to be anticipated for instruction, and to consider their suitability as points of impact for a process of progressive mathematisation. The goals of phenomenological investigation are to find problem situations in which situation-specific approaches can be generalized, and to find situations that can evoke paradigmatic solution procedures which can be taken as the basis for vertical mathematisation (Freudenthal, 1983 as cited in Gravemeijer, 1997, p. 329).

3. Self-developed model: A self-developed model plays a vital role in bridging the gap between informal knowledge and formal mathematics. Models were developed by the students themselves. At first, the model was any model of a situation familiar to the students. By generalizing and formalizing, the model then becomes an entity of its own. This made it possible to use this model as a model for mathematical reasoning (Gravemeijer, 1994 as cited in Gravemeijer, 1997, p. 329).

DAPIC Problem-solving Process

DAPIC (Define – Assess – Plan – Implement – Communicate) is a problem-solving process developed and employed as an integral part in the Integrated Mathematics, Science, and Technology (IMaST) Program, a mathematics, science, and technology education curriculum designed for the middle grades and developed by Illinois State University's Center for Mathematics, Science, and Technology (CeMaST) with grants from the National Science Foundation, Eisenhower funds from the Illinois State Board of Education, and Illinois State University (Center for Mathematics, Science, and Technology [CeMaST], 1998).

The DAPIC problem-solving process is based on Polya's mathematical model—the science method of inquiry—and Shewhart's Cycle of industrial problem solving (Meier, Hovde, & Meier, 1996, p. 234). The five components of the DAPIC problem-solving process are described as follows (Meier et al., 1996, p. 235).

1. Define (D): The problem is identified. This may require asking questions, collecting some preliminary data, learning some new vocabulary or factual material. The problem is usually defined from students' experiences.
2. Assess (A): The problem situation is assessed and information is collected. Data is used to make a generalization in the form of a hypothesis that may require some additional investigation before the main investigation takes place.
3. Plan (P): A plan is established to solve the problem and to collect data. This often means using an experimental design in which variables are controlled.
4. Implement (I): Carry out the plan. Data is collected and analyzed based on the plan, making modifications as the need arises.
5. Communicate (C): Results are analyzed and evaluated, as well as shared with others. Results are assessed for accuracy and relevance. This is done in the form of written or oral reports on the project consequences and to look forward to possible subsequent investigations.

Figure 1 shows that DAPIC can be visualized as a loop with multiple entry points, having no obvious starting point or order. DAPIC does not become too linear. Some parts may be omitted, added, or repeated. The order may not always be the same. Teachers must be certain that students have an opportunity to use DAPIC in a variety of ways and enter the process at various points. In the IMaST curriculum, DAPIC is a tool used to help learn other concepts, as well as an outcome itself (CeMaST, 1998, pp. 10–11; Meier et al., 1996, pp. 235–236).

Conceptual Frameworks

From the problem and the study of theoretical background, the researcher sets conceptual frameworks for this study to develop an instructional process for enhancing mathematical literacy through realistic mathematics education and the DAPIC problem-solving process as shown in Figure 2.

Methods

This study consists of two phases: Phase 1 the development of the instructional process and Phase 2 the experiment of the developed instructional process.

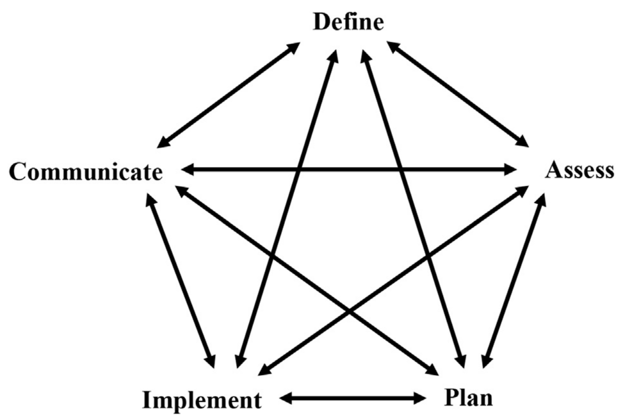

Figure 1 Interaction in the DAPIC problem-solving process
Source: Meier et al. (1996, p. 236)

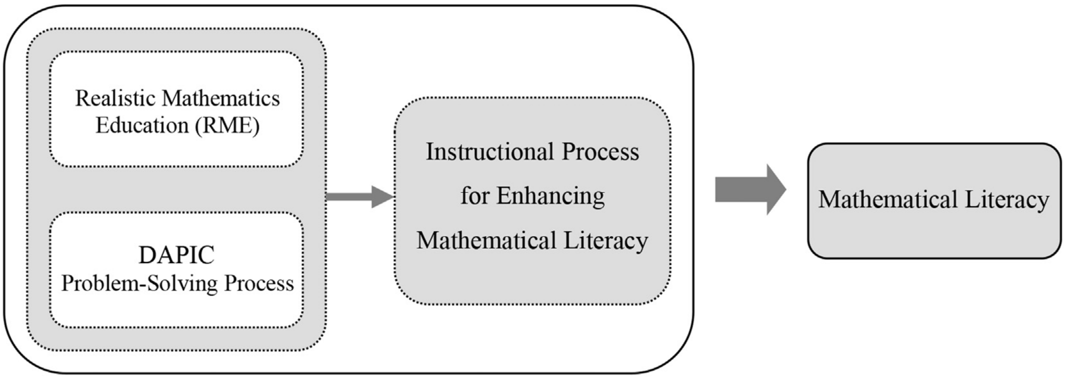

Figure 2 Conceptual frameworks

Phase 1: Development of the Instructional Process

Regarding the development of an instructional process for enhancing secondary school students' mathematical literacy by using realistic mathematics education and DAPIC problem-solving process, the researcher determined the instructional process development framework as shown in Figure 3.

The details of instructional process development were as follows:

1. The substantive analysis of realistic mathematics education and the DAPIC problem-solving process as the principles for instructional process development:

There were several key substances of realistic mathematics education:

1) Problems or situations occurring in real life were used as a starting point for learning and development of the mathematics concept;

2) Mathematics learning should enable students to reinvent mathematics under the teachers' guided reinvention;

3) Students were promoted to develop and to use the simple self-developed method to solve problems, and then further develop formal mathematics; and

4) Discussing and interacting in the classroom were important for developing mathematical knowledge.

There were several key substances of the DAPIC problem-solving process as it is a mathematical and scientific problem-solving process which could be used to solve problems occurring both inside and outside classrooms, as well as problems related to real life. There were five key elements as follows:

1) Define: to determine or define problem clearly;

2) Assess: to assess problem situation;

3) Plan: to plan how to solve the problem;

4) Implement: to implement the desired plan and to develop the plan more appropriately; and

5) Communicate: to analyze and to evaluate the implementation outcomes, as well as to communicate the results to others.

2. The creation of instructional process principles: the researcher applied the substances of realistic

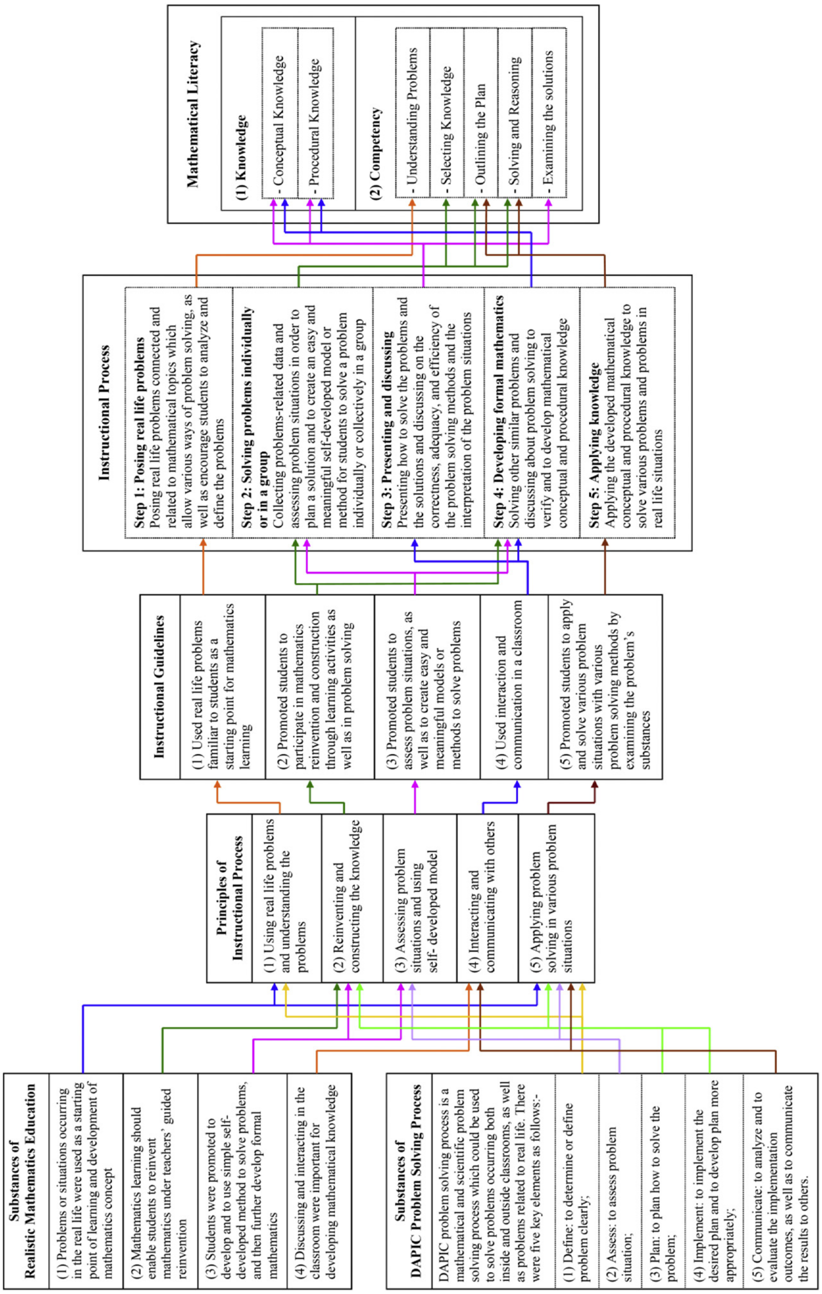

Figure 3 Framework of instructional process

mathematics education and the DAPIC problem-solving process and then integrated them as the principles of realistic mathematics education and the DAPIC problem-solving process for use in the instructional process.

The principles of realistic mathematics education and the DAPIC problem-solving process which were also used as the principles of instructional process for enhancing secondary school students' mathematical literacy consisted of five key elements as follows:

1) ***Principle of using real life problems and understanding the problems***

Using real life problems which students were familiar with as a starting point for mathematics learning could encourage students to have better understanding of problems and to create more meaningful learning from them;

2) ***Principle of reinventing and constructing the knowledge***

Mathematics learning was an activity for constructing knowledge instead of transmitting the existing knowledge. Students should participate in reinventing mathematics through learning activities;

3) ***Principle of assessing problem situations and using self-developed model***

Students should assess problem situations, as well as create easy and meaningful models or methods to solve the problems. A self-developed model or method should be developed into a more formal procedure;

4) ***Principle of interacting and communicating with others***

Interacting and communicating in the classroom could encourage students to verify and to develop mathematical ideas;

5) ***Principle of applying problem solving in various problem situations***

Students should apply and solve various problem situations by implementing various problem-solving methods. Examining problem's substances could encourage students' problem-solving abilities.

3. Analysis of the instructional guidelines to enhance students' mathematical literacy: the researcher used the principles of instructional process to analyze and to develop the instructional guidelines to enhance students' mathematical literacy.

The instructional guidelines to enhance secondary school students' mathematical literacy based on realistic mathematics education and the DAPIC problem-solving process could be summarized as follows:

1) Used real life problems familiar to students as a starting point for mathematics learning to encourage students to have better understanding on problems and to create more meaningful learning from them;

2) Promoted students to participate in mathematics reinvention and construction through learning activities as well as in problem solving to enhance their better understanding of mathematical concepts and procedures;

3) Promoted students to assess problem situations, as well as to create easy and meaningful models or methods to solve problems to enhance students' efficiency in using and selecting a model or method for problem solving;

4) Used interaction and communication in the classroom to encourage students to verify and to develop mathematical ideas and problem-solving abilities; and

5) Promoted students to apply and solve various problem situations with various problem-solving methods by examining the problem's substances to enhance students' problem-solving abilities.

4. Determination of instructional process: the researcher used the instructional guidelines to synthesize an instructional process.

An instructional process to enhance secondary school students' mathematical literacy based on realistic mathematics education and the DAPIC problem-solving process could be explained as follows.

Step 1: Posing real life problems

This step focused on posing real life problems connected and related to mathematical topics which allow various ways of problem solving, as well as encouraging students to analyze and define the problems.

Instructional Activities

1. A teacher designs and presents a problem occurring in a real life situation to review existing knowledge which is necessary to learn new knowledge. Then, a teacher guides students to solve such problems by using their familiar or experienced methods, and to lead them to learn new knowledge;
2. A teacher designs and presents a problem occurring in a real life situation related to mathematical topics which a teacher plans to teach by using pictures, stories, diagrams, or symbols familiar to the students. A problem could be solved by various methods;
3. Students analyze and try to understand the problem and then determine or define the problem more clearly.

Step 2: Solving problems individually or in a group

This step focused on collecting problems-related data and assessing problem situations in order to plan a solution and to create an easy and meaningful self-developed model or method for students to solve a problem individually or collectively. The teachers' roles were being facilitators encouraging students to use various strategies and heuristics to solve problems or guiding them when they were facing difficulty during the problem-solving process.

Instructional Activities

1. Students collect problems-related data and assess problem situations in order to plan how to solve a problem;
2. Students invent and create a self-developed model or method to solve a problem by applying their existing experience or familiar methods;
3. Students solve a problem individually or collectively;
4. A teacher guides students on strategies and heuristics to solve the problem, i.e. drawing pictures on blackboard and advising students individually or collectively upon request.

Step 3: Presenting and discussing

This step focused on presenting and discussing how to solve the problems and the solutions which lead to the examination of various problem-solving methods. The discussion focused on the correctness, adequacy, and efficiency of the problem-solving methods and the interpretation of problem situations. During this step, students had to compare and justify the solutions and problem-solving methods with others.

Instructional Activities

1. A teacher lets students present their own or their group's problem-solving methods and their decided solutions to the class;
2. A teacher conducts a discussion for students to exchange their views on the correctness, adequacy, and efficiency of various problem-solving methods, as well as the interpretation of problem situations;
3. Students participate in such discussion by comparing their solutions with their classmates' solutions, as well as communicating, arguing about, and judging their own solutions.

Step 4: Developing formal mathematics

This step focused on solving other similar problems and discussing problem-solving methods which would lead to the formulation of solution-finding procedures. In this step, there were several discussions among students or between students and teachers to verify and to develop mathematical conceptual and procedural knowledge.

Instructional Activities

1. A teacher designates several problems occurring in real life situations (which could be solved with similar problem-solving methods) for students to solve;
2. Students solve problems individually or collectively;
3. A teacher encourages students to develop more formal problem-solving methods and mathematical languages through discussion;
4. A teacher and students collaborate in such discussion to verify and develop mathematical conceptual and procedural knowledge;
5. A teacher and students collaboratively conclude mathematical conceptual and procedural knowledge.

Step 5: Applying knowledge

This step focused on applying the developed mathematical conceptual and procedural knowledge to solve various problems and problems in real life situations.

Instructional Activities

1. A teacher designates various problems and problems in real life situations for students to apply the developed mathematical conceptual and procedural knowledge;
2. Students examine problems' substances and selectively apply mathematical conceptual and procedural knowledge which are suitable for each problem;
3. A teacher guides and facilitates students upon request.

Phase 2: Experiment of the Developed Instructional Process

The pre-test/post-test control group method was used to appraise the experimenting effectiveness of the developed instructional process on mathematical literacy. The sample group consisted of 104 ninth grade students from a secondary school in Bangkok, Thailand (52 students for an experimental group and another 52 students for a control group). The experiment was conducted over a period of 15 weeks (45 h).

Research instruments

1. ***Mathematical Literacy Tests (Knowledge)*** consisted of 30 multiple choice items. Pre-test and post-test mathematical literacy (Knowledge) were equivalent and used to assess mathematical conceptual and procedural knowledge, including surface area and volume, graphs of linear relationship, and two-variable linear equation systems. Both tests were verified by three experts in mathematical teaching and trialed (p = .227–.795, r = .213–.679 and Cronbach alpha (reliability) = .762).
2. ***Mathematical Literacy Tests (Competency)*** consisted of five real life problems requiring students to apply mathematical conceptual and procedural knowledge for problem solving. Each problem consisted of five questions requiring students to (1) understand problems, (2) select knowledge, (3) outline the plan, (4) solve and provide reasoning, and (5) examine the solutions. Mathematical Literacy Tests (Competency) and scoring rubrics were verified by three experts in mathematical teaching and trialed.

2.1 *Pre-test of mathematical literacy (Competency)* was used to assess the competency in applying knowledge on the topics of Pythagorean Theorem, real numbers, and one-variable linear equations that students had studied in the previous semester prior to the experiment (p = .249–.720, r = .209–.557 and Cronbach alpha (reliability) = .748).

Table 1
Comparisons of mathematical literacy between the experimental group and the control group after the experiment

Mathematical literacy After the experiment	Group	n	Mean	s	df	t	*p*
Mathematical literacy (Knowledge)	Experimental	52	22.442	2.789	102	5.190	.000
	Control	52	19.423	3.133			
Mathematical literacy (Competency)	Experimental	52	32.731	8.003	102	10.320	.000
	Control	52	16.865	7.672			

$p < .05$

Table 2
Comparisons of mathematical literacy of the experimental group before and after the experiment

Mathematical literacy (Knowledge) The experimental group	n	Before the experiment		After the experiment		df	t	*p*
		Mean	s	Mean	s			
Mathematical literacy (Knowledge)	52	12.135	3.138	22.442	2.789	51	27.858	.000
Mathematical literacy (Competency)	52	17.135	7.844	32.731	8.003	51	13.689	.000

$p < .05$

2.2 *Post-test of mathematical literacy (Competency)* was used to assess the competency in applying knowledge on the topics of surface area and volume, graphs of a linear relationship, and a two-variable linear equation system which were used in the experiment (p = .262−.743, r = .243−.569 and Cronbach alpha (reliability) = .754).

Procedure

The researcher taught the students in the experimental group through lesson plans based on the developed instructional process (posing real life problems, solving problems individually or in a group, presenting and discussing, developing formal mathematics, and applying knowledge), whereas students in the control group were taught using traditional lesson plans, for 15 weeks (45 h). Both groups were taught on the topics of surface area and volume, graphs of linear relationships, and two-variable linear equation systems. Mathematical Literacy Tests were used both pre-test and post-test. During the experiment, the researcher observed the realistic problem-solving behavior of students in the experimental group and assessed students' realistic problem solving. Students conducted self-assessment at the end of the 5th, 10th, and 15th weeks. A questionnaire regarding the instructional process was used to survey students' opinions at end of the experiment.

Findings

The mathematical literacy of the experimental group students instructed through developed instructional process (posing real life problems, solving problems individually or in a group, presenting and discussing, developing formal mathematics, and applying knowledge), was significantly higher than pre-learning and higher than those of the control group students. The experimental group students were able to solve real life mathematical problems effectively as follows:

1. The mathematical literacy of students in the experimental group and the control group after the experiment is shown in Table 1.

As shown in Table 1, the mathematical literacy of students in the experimental group after learning through the developed instructional process were significantly higher than the control group in both knowledge and competency at the .001 level of significance.

2. The mathematical literacy of students in the experimental group before and after the experiment is shown in Table 2.

As shown in Table 2, the mathematical literacy of students in the experimental group after learning through the developed instructional process was significantly higher than those before learning in both knowledge and competency at the .001 level of significance.

Discussion and Recommendation

Based on the findings, the mathematical literacy of the experimental group students was higher than that of the control group, which confirmed that the collaborative use of realistic mathematics education and the DAPIC problem-solving process could enhance students' mathematical literacy. This was due to the principles of the instructional process for enhancing students' mathematical literacy based on realistic mathematics education and the DAPIC problem-solving process which consisted of several elements: (1) using real life problems with which students were familiar as a starting point for learning mathematics could enhance better understanding of the problems and make the learning more meaningful; (2) promoting students to participate in the reinvention and construction of mathematics through learning activities and problem solving could enhance better understanding of mathematical concepts and procedures; (3) promoting students to assess problem situations and to create easy and meaningful models or methods for problem solving could enhance students' efficiency in using and selecting a model or method for problem solving; (4) using interaction and communication in the classroom could help them in verifying and developing mathematical ideas; and (5) promoting students to apply and solve various problem situations with various problem-solving methods by examining the problem's substances could enhance students' problem-solving abilities.

A teacher can apply the five steps of the developed instructional process to enhance the mathematical literacy of students in secondary school. A teacher should analyze students' backgrounds and choose problems related to their background in order to promote students' understanding of problems and finding solutions. In such a process, a teacher should be patient and allow students to develop a solution procedure by themselves and the teacher can help to facilitate using guided heuristics if necessary.

Conflict of Interest

There is no conflict of interest.

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